Review Q & A - Oct. 20

Written Test 1

Practice Test Questions
Math Review Lecture

Written Test 1: Practice Question 1

Consider the following predicate:

! x, y . x : NAT & y: NAT => x * y > 0

$\forall x, y \cdot x \in \mathbb{N} \land y \in \mathbb{N} \Rightarrow x * y > 0$

T or F

Consider each of the following statements **in isolation**, choose **all** that are correct.

- [ ] 1. The predicate is not a theorem and can be disproved by an x-y pair (5, 0).
- [ ] 2. The predicate is not a theorem and can be disproved by an x-y pair (12, 13).
- [ ] 3. The predicate is a theorem and can be proved by an x-y pair (2, 3).
- [ ] 4. The predicate is not a theorem and can be disproved by an x-y pair (12, -2).
- [ ] 5. None of the listed statements is correct.
- [ ] 6. The predicate is a theorem and can be proved by an x-y pair (5, 4).

$T \Rightarrow F \equiv F$

$T \Rightarrow T \equiv T$

witness: $5 \in \mathbb{N} \land 0 \in \mathbb{N} \Rightarrow 5 * 0 > 0$

witness: $12 \in \mathbb{N} \land 13 \in \mathbb{N} \Rightarrow 12 * 13 > 0$

witness: $12 \in \mathbb{N} \land -2 \in \mathbb{N} \Rightarrow 12 * -2 > 0$

$F \Rightarrow \_ \equiv T$

T

Written Test 1: Practice Question 2

Consider the following predicate:

#x, y . x : NAT & y: NAT & x * y > 0

$\exists x, y \cdot \boxed{x \in \mathbb{N} \land y \in \mathbb{N} \land x * y > 0}$

Consider each of the following statements **in isolation**, choose **all** that are correct.

- [ ] 1. The predicate is not a theorem and can be disproved by an x-y pair (5, 0).
- [ ] 2. The predicate is not a theorem and can be disproved by an x-y pair (12, -2).
- [ ] 3. None of the listed statements is correct.
- [ ] 4. The predicate is a theorem and can be proved by an x-y pair (-2, -3).
- [ ] 5. The predicate is a theorem and can be proved by an x-y pair (5, 4).
- [ ] 6. The predicate is a theorem and can be proved by an x-y pair (2, 3).
- [ ] 7. The predicate is not a theorem and can be disproved by an x-y pair (12, 13).

① to prove $\exists$, need one witness

② to disprove, need to consider all combinations.

$\underbrace{-2 \in \mathbb{N} \land -3 \in \mathbb{N}}_{F} \land \underbrace{-2 * -3 > 0}$ F

$\underline{5 \in \mathbb{N}} \land \underline{4 \in \mathbb{N}} \land \underline{5 * 4 > 0}$ T

F $\boxed{\underbrace{5 \in \mathbb{N}}_{T} \land \underbrace{0 \in \mathbb{N}}_{T} \land \underbrace{5 * 0 > 0}_{F}}$

this witness evaluates to false, but not sufficient to disprove $\exists$.

Written Test 1: Practice Question 3

precedence

$\{a, b, c, d\} \setminus \{a, e\} \cup \{a, f\}$

Given two sets S and T, say we write:

- S \/ T for their union
- S /\ T for their intersection
- S \ T for their difference

What is the **cardinality** of the power set of ({a, b, c, d} \ {a, e}) \/ {a, f}? Enter an integer value (with no spaces).

Answer: 32

$$|\mathbb{P}((\{a, b, c, d\} \setminus \{a, e\}) \cup \{a, f\})|$$

$$\{b, c, d\} \cup \{a, f\}$$

$$|\mathbb{P}(\{a, b, c, d, f\})|$$

$$2^{|\{a, b, c, d, f\}|} = 2^5 = 32$$

Written Test 1: Practice Question 4

precedence

$\neg$

$\wedge$

$\vee$

$\Rightarrow$

Consider the following logical quantification:

!x,y.x:NAT&y:NAT=>x+y>=10&x+y<20

Convert the above predicate to an equivalent one using the other logical quantifier.

$\forall x \cdot R(x) \Rightarrow P(x)$

$\equiv \neg \exists x \cdot R(x) \wedge \neg P(x)$

Note the following constraints on your answer:

- Only put pairs of parentheses **when necessary**.
- Like the above predicate, there should be **no** white spaces.
- Like the above predicate, numerical constants (i.e., 10, 20) must appear as the right operands of the relational expressions (e.g., x + y >= 10).
- Relational expressions should be simplified whenever possible, e.g., write x >= 20 rather than not(x < 20).

$p \wedge q \vee r \equiv (p \wedge q) \vee r$

Be cautious about the spellings: this question will be graded **automatically** and no partial marks will be give to spelling mistakes.

Answer:

$! \, x, y \, . \, \underbrace{\boxed{x : NAT \,\&\, y : NAT}}_{R(x,y)} \Rightarrow \underbrace{\boxed{x + y >= 10 \,\&\, x + y < 20}}_{P(x,y)}$

not (x+y >=10 & x+y<20)

not (# x,y . x : NAT & y : NAT & (x+y<10 or x+y >= 20))

$\equiv$ $\boxed{\text{not} (x + y >= 10)}$ or $\boxed{\text{not} (x + y < 20)}$

x+y<10

x+y >= 20

Written Test 1: Practice Question 5

Consider two sets:

- S = {x, y}
- T = {1, 2, 3}

$S \times T$

Write out the **maximum** relation r such that r : S <-> T.

$r \in \boxed{S \leftrightarrow T}$ the set of all possible relations between S and T.

Requirements. In your answer:

- Pairs must be **sorted** in an **ascending** order by the first elements, or by the second elements if the first elements are identical. For examples: (x, 2) appears before (y, 1), (x, 1) appears before (x, 2), etc.
- No white spaces should be included, e.g., write (x,1) rather than (x, 1).

Be cautious about the spellings: this question will be graded **automatically** and so no partial marks will be given due to spelling mistakes.

Answer:

{(x,1),(x,2),(x,3),(y,1),(y,2),(y,3)}

Written Test 1: Practice Question 6

Consider two sets:

- S = {x, y}
- T = {1, 2, 3}

Enumerate the following set:

$\boxed{\{(a,b) \mid a : S \;\&\; b : T \;\&\; a \;/\!=\; x \;\&\; b < 3\}}$

Q. $\{(a,b) \mid a \in S \wedge b \in T \wedge a = x \wedge \boxed{b > 3}\} = \{\}$ ✓ range

property

$\{(a,b) \mid \boxed{a \in S \wedge b \in T} \wedge \boxed{a \neq x} \wedge \boxed{b < 3}\}$

$\{\cancel{(x,1)}, \cancel{(x,2)}, \cancel{(x,3)}, (y,1), (y,2), \cancel{(y,3)}\}$

Requirements. In your answer:

- Pairs must be **sorted** in an **ascending** order by the first elements, or by the second elements if the first elements are identical. For examples: (x, 2) appears before (y, 1), (x, 1) appears before (x, 2), etc.
- No white spaces should be included, e.g., write (x,1) rather than (x, 1).

Be cautious about the spellings: this question will be graded **automatically** and so no partial marks will be given due to spelling mistakes.

Answer: $\{(y,1), (y,2)\}$

Property: ① $a \neq x \vee b < 3$ ③ $\neg(a \neq x \vee b < 3)$

(Exercises) ② $a \neq x \Rightarrow b < 3$

Written Test 1: Practice Question 7

$\binom{n}{m}$ Given n elements, how many ways can we make subsets of size m?

Consider two sets:

- S = {x, y}
- T = {1, 2, 3}

<| dom. res.

<<| dom. sub.

Consider r such that r : S <-> T:

{(x, 1), (x, 3), (y, 1), (y, 2)}

$r = \{(x,1), (x,3), (y,1), \cancel{(y,2)}\}$

What is the result of the following expression:

{x} <<| (r |> (T \ {2}))

ran. res.

dom ~~res.~~ sub.

Requirements. In your answer:

- Pairs must be **sorted** in an **ascending** order by the first elements, or by the second elements if the first elements are identical. For examples: (x, 2) appears before (y, 1), (x, 1) appears before (x, 2), etc.
- No white spaces should be included, e.g., write (x,1) rather than (x, 1).

Be cautious about the spellings: this question will be graded **automatically** and so no partial marks will be given due to spelling mistakes.

ex. $\{x\} \lhd (r \rhd (T \setminus \{2\}))$

Answer:

$\{x\} ⩤ (r \rhd \boxed{(T \setminus \{2\})})$

$\{1, 3\}$

$\{\cancel{(x,1)}, \cancel{(x,3)}, (y,1)\}$

$\{(y,1)\}$

e.g. $\left|\{ s \mid s \in \mathbb{P}(\{1,2,3,4,5,6\}) \wedge (|s| = 3 \vee |s| = 5)\}\right|$

$= \binom{6}{3} + \binom{6}{5}$

$= 20 + 6$

$= \boxed{26}$.

Compute: $\mathbb{P}(\{1, 2, 3, 4, 5, 6\})$

$$\left\{ \begin{array}{ll} \emptyset & \text{subset of size } 0 \\ \vdots & \\ \boxed{} & \boxed{\text{subsets of size } 3} \ \binom{6}{3} \\ \vdots & \\ \{1,2,3,4,5,6\} & \text{subset of size } 6 \end{array} \right\}$$

$\binom{6}{5} = \binom{6}{1} = 6$

$\binom{6}{3} = \frac{6 * 5 * 4}{3!} = \boxed{20}$